

TRIGONOMETRY PROBLEM SOLVING
Math

Grades: 9-12

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Acceleration Approach

Standard has been accelerated by moving grade level 9 up to the standard used for grade level CIM.

1	2	3	4	5	6	7	8	9	CIM	CRLS/ CAM	PASS
								—————▶			

Organizing Overarching Concept (e.g. systems, patterns of change, models, scales)

Models: Indirect Measurement

Organizing Higher Order Skills (e.g. Bloom’s, Paul’s Model of Reasoning)

Bloom’s Analyzing and Evaluation

Differentiation Features – Students:

- Use advanced resources
- Use sophisticated content stimuli
- Make reasoning explicit

Implementation Time

1 week

COMMON CURRICULUM GOAL

Geometry—Properties and Relationships

Analyze characteristics and properties of two and three dimensional geometric shapes and develop mathematical arguments about geometric shapes.

GRADE LEVEL STANDARDS

Use trigonometric functions to solve for an unknown length of a side of a right triangle given an angle and a length of a side (assessed on state knowledge and skills test)

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TASK DEMAND

Advanced problem solving using trigonometry
Although the Earth is not a sphere, it can be treated as though it was spherical for many purposes.

Questions

1. Show that the length of any parallel of latitude around the Earth is equal to the equatorial distance around the Earth (its circumferences) times the cosine of the latitude angle q .
2. The radius of the Earth is about 6400 km. What is the length of the 30° parallel, north or south latitude?
3. Two tracking stations S mile apart measure the elevation angle of a weather balloon to be a and b . What is a formula for the altitude h of the balloon in terms of a , b , S and the cotangent function (Ignore Earth's curvature)?
4. A satellite traveling in a circular orbit 1600 km above the Earth is due to pass directly over the tracking station at noon. Assume that the satellite takes two hours to make an orbit and that the Earth's radius is 6400 km.
 - a. If the tracking antenna is aimed 30° above the horizon, at what time will the satellite pass through the beam of the antenna?
 - b. Find the distance between the satellite and the tracking station at 12:03 pm.
 - c. At what angle above the horizon should the antenna be pointed so that its beam will intercept the satellite at 12:03 pm?
5. Two NASA tracking stations are located near the equator; one is in Ethiopia at 40° east longitude, another near Quito, Ecuador at 78° west longitude. Assume that both stations, denoted E and Q are on the equator and that the radius of the Earth is 6400 km. A satellite in orbit over the equator is observed at the same instant from both tracking stations. The angles of elevation above the horizon are 5° from Quito and 10° from Ethiopia. Find the distance of the satellite from Earth at the instant of observation.

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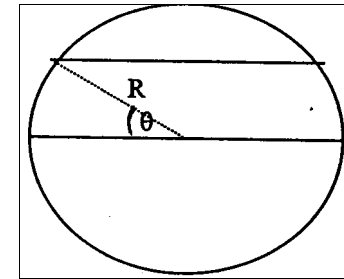
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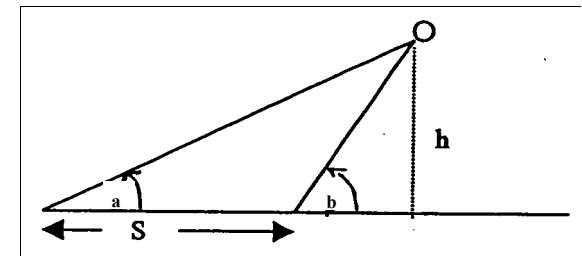
Sample Task Activity #1

1. Show that the length of any parallel of latitude around the Earth is equal to the equatorial distance around the Earth (its circumference) times the cosine of the latitude angle q :
2. The radius of the Earth is about 6400 km. Find the length of the 30° parallel, north or south latitude.



Sample Task Activity #2

1. Two tracking stations S mile apart measure the elevation angle of a weather balloon to be a and b . Derive a formula for the altitude h of the balloon in terms of a , b , S and the cotangent function (Ignore Earth's curvature).



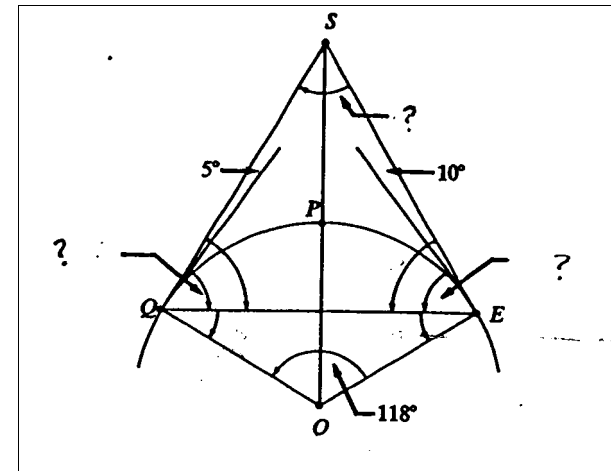
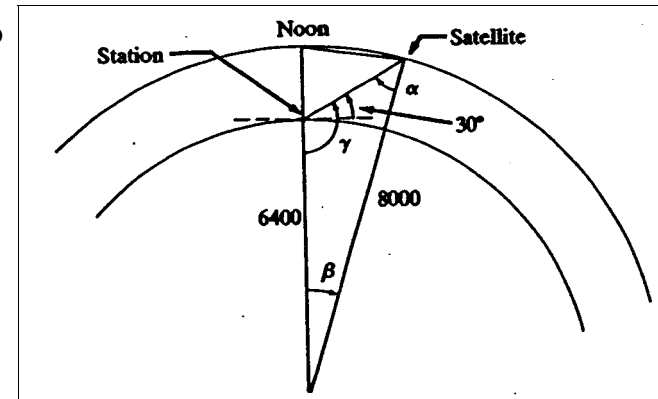
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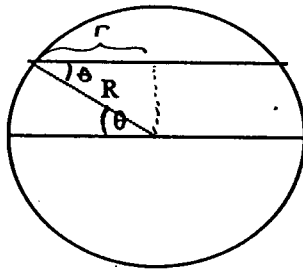
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Solutions

1. Show that the length of any parallel of latitude around the Earth is equal to the equatorial distance around the Earth (its circumference) times the cosine of the latitude angle, θ :



$$r = R \cos \theta$$

then distance around Earth
is $2\pi r = 2\pi R \cos \theta$

But $2\pi R =$ Earth's
circumference, C_E
so

$$2\pi r = C_E \cos \theta$$

call this C_p , circumference at
parallel.

2. The radius of the Earth is about 6400km. Find the length of the 30° parallel, north or south latitude.

$$C_p = 2\pi (6400\text{km})(\cos 30) = 5500 \text{ km}$$

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Solutions



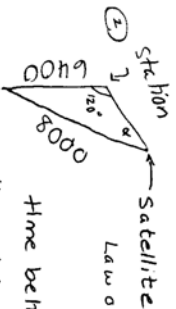
$\cot \beta = x/h \Rightarrow x = h \cot \beta$



$\cot \alpha = \frac{S+x}{h}$

$\Rightarrow h \cot \alpha = S + h \cot \beta$

so $h = \frac{S}{\cot \alpha - \cot \beta}$



Law of Sines:

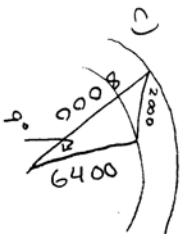
$\sin \alpha = \frac{6400 \cdot \sin(120^\circ)}{8000} = .693$ so $\alpha = 44^\circ$ and $\beta = 16^\circ$

Time between $\beta = 16^\circ + \beta = 0^\circ$ is $\frac{16^\circ}{360^\circ} (120 \text{ min}) = 5.3 \text{ minutes}$
So, satellite will pass through beam @ $11:54.7 \text{ a.m.}$

b.) $\beta = \frac{3 \text{ min}}{120 \text{ min}} (360^\circ) = 9^\circ$

Use Law of Cosines:

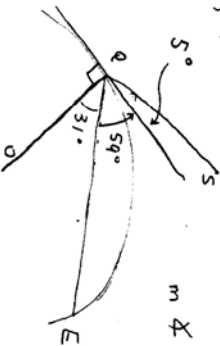
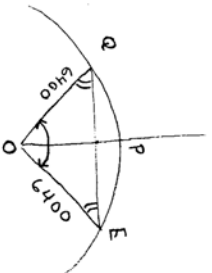
$X^2 = 6400^2 + 8000^2 - 2(6400)(8000) \cos 9^\circ$
 $X = 1960$ or about 2000 km



Law of Sines: $\frac{\sin 9^\circ}{2000} = \frac{\sin(\alpha + 90^\circ)}{8000}$

$\alpha = 51^\circ$

③



$m \angle QOE = 78 - (-40) = 118^\circ$. ΔQOE is isosceles
So, $\angle OQE \cong \angle OEQ$ and each measures 31°

$m \angle SQE = 59^\circ + 5^\circ = 64^\circ$

likewise,
 $m \angle SEO = 69^\circ$
and thus
 $m \angle QSE = 47^\circ$

We are looking for the measure of \overline{SP} . To do so, find the measure of \overline{OS} and just subtract 6400 km .

1st find $\angle QE$: $\frac{\angle QE}{\sin 118} = \frac{OE}{\sin 31}$ $QE = 10940 \text{ km}$

then find SE : $\frac{SE}{\sin 64} = \frac{QE}{\sin 47}$ $SE = 13400 \text{ km}$

and $OS^2 = OE^2 + SE^2 - 2(OE)(SE) \cdot \cos(\angle OES) \Rightarrow OS = 15,800 \text{ km}$

Thus $SP = 9400 \text{ km}$

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TAG NEEDS ADDRESSED

INTELLECTUALLY GIFTED	ACADEMICALLY TALENTED MATH	CAREER RELATED LEARNING STANDARDS FOR CAM - Certificate of Advanced Mastery	TEACHER CHECKS THE BENCHMARK LEVEL STUDENT
<input checked="" type="checkbox"/> Advanced Critical Reasoning <input type="checkbox"/> Scholarly Interaction <input checked="" type="checkbox"/> Continuous Progress for Level and Rate* <input type="checkbox"/> Challenging Resources <input type="checkbox"/> Effecting Change <input type="checkbox"/> Decision Making; Ethical Use of Influence <input type="checkbox"/> Leadership Training/Career <input type="checkbox"/> Realistic Goal Setting <input type="checkbox"/> Regular Interaction with Intellectual Peers <input type="checkbox"/> Social-Emotional Issues; Support; Coping Strategies <input type="checkbox"/> Advanced Academic Planning <input type="checkbox"/> Opportunity for Competition/ Failures/Successes <input type="checkbox"/> Creative Problem Solving with Real Problems/Audience <input type="checkbox"/> Pursuit of Advanced Level Research <input type="checkbox"/> Advanced Vocabulary Development	<input checked="" type="checkbox"/> Advanced Creative Thinking in Math <input checked="" type="checkbox"/> Continuous Progress/Level and Rate* in Math <input type="checkbox"/> Challenging Math Resources <input type="checkbox"/> Creative Problem Solving Strategies in Math <input type="checkbox"/> Math Advanced Vocabulary Development <input type="checkbox"/> Leadership Training/Career <input type="checkbox"/> Decision Making; Ethical Use of Influence <input type="checkbox"/> Regular Interaction with Talented Math Peers <input type="checkbox"/> Realistic Goal Setting <input type="checkbox"/> Opportunity for Competition/Failures/ Successes <input type="checkbox"/> Advanced Academic Planning in Math	<input type="checkbox"/> Personal Management <input checked="" type="checkbox"/> Problem Solving <input type="checkbox"/> Communication <input type="checkbox"/> Teamwork <input type="checkbox"/> Organizations and Systems <input type="checkbox"/> Employment Foundations <input type="checkbox"/> Career Development <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>* Rate requires monitoring to ensure that the student was allowed to move ahead upon acquiring concepts.</p> </div>	<p>Math:</p> <input type="checkbox"/> 1 <input type="checkbox"/> 2 <input type="checkbox"/> 3 <input type="checkbox"/> CIM <input type="checkbox"/> CAM
<p>Student _____ Grade _____</p> <p>Teacher _____ School _____</p> <p>Date Initiated _____ Date Completed _____</p> <p style="text-align: center;">Check TAG Identification category: <input type="checkbox"/> Intellectual <input type="checkbox"/> Academic Math <input type="checkbox"/> Academic LA</p>			