DIFFERENTIATED CURRICULUM CHALLENGING HIGH END LEARNERS ADDRESSING OREGON STANDARDS & BENCHMARKS

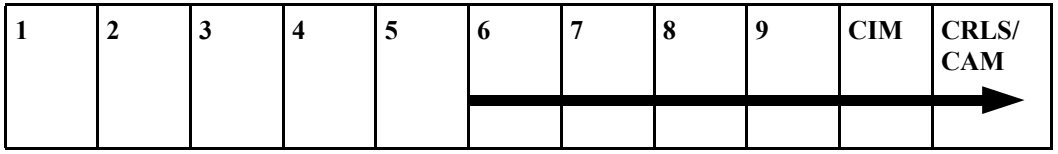
SPACE MATHEMATICS - ALGEBRA

Math

Grades: 6-8

Acceleration Approach

Standard has been accelerated by moving grade level 6 up to the standard used for grade level CIM.



Students solve problems that arise in the study of space. Emphasis is on algebra.

Task 1: Telescopes allow astronomers to make observations and compare them to cosmological theories about the age and formation of the universe. It all began when Galileo introduced the telescope in 1609: this revolutionized astronomy. In more recent times, NASA has launched space telescopes above the Earth's atmosphere so that scientists are able to see much fainter objects and thus gather much more detailed data.

1. The space telescope is able to see starts and galaxies whose brightness is only one-fiftieth of the faintest object observable from Earth. The brightness of a point source such as a star varies inversely with the square of its distance from the observer. How much further into the universe can the space telescope see compared to the ground-based telescopes?

The parsec is the astronomical unit of distance that relates to observational measurements. 1 parsec = 3.09×10^{13}km.

2. Light (photons) travel at a rate of 2.99×10^{9} meters every second. A light year is the distance light travels in a vacuum in one year. How many light years are there in one parsec?

3. Because of the time it takes for light to travel from distance starts and galaxies, we see them as they were some time ago.
   a. The best ground-based telescopes can see objects about 10^{9} parsecs from our solar system. How long ago were the photons emitted that we now see when we observe such an object?
   b. When the space telescope makes an observation, how far back in time will it see stars and galaxies?

Task 2: Kepler and Newton

Johann Kepler was a masterful astronomer/mathematician who derived three laws that govern the movement of the planets around the sun. Kepler's third law relates the period (time for one complete revolution) of two planets to their average distances from the sun. If we represent the periods of any two planets by T and t and their average distances

COMMON CURRICULUM GOAL

Mathematics—Algebraic Relationships
Represent and analyze mathematical situations and structures using algebraic symbols.

Mathematics—Statistics
Select and use appropriate statistical methods to analyze data.

GRADE LEVEL STANDARDS

CIM—Write and evaluate algebraic equations involving absolute value, rational exponents, and roots.

CIM—Determine the mean and standard deviation of a normal distribution and interpret results.

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from the sun by R and r, respectively, then his law may be written:

$$\left(\frac{T}{t}\right)^2 = \left(\frac{R}{r}\right)^3$$

Newton was a brilliant mathematician/physicist who developed many important laws. In fact, Newton used Kepler's third law (above) to develop the inverse square law of gravity. If two bodies are a distance r apart, then the gravitational force F_g between them is given by:

$$F_g = \frac{K}{r^2}$$

where K is a constant that depends on the masses of the two bodies. Follow these steps to do what Newton did: derive the law of gravity using Kepler's third law:

Step 1: Start with Kepler's third law. Assume you know the values of *t* and *r*. Replace them with a constant A, so that $\left(\frac{T}{t}\right)^2 = \left(\frac{R}{r}\right)^3$ is in the form: $T^2=AR^3$ What is the value of A?

Step 2: Assume the planets move in a circular orbit. Remember that velocity is distance divided by time. Find the velocity of a planet moving in a circular orbit of radius r with a period of *T*.

Step 3: Why don't the planets stay in orbit rather than flying off into space? Because the gravitational pull from the sun keeps them in their orbits. For a body of mass m moving in a circular path of radius *r*, the inward (centripetal) force that keeps them in a circle is given by

$$F = \frac{mv^2}{r}$$

Use your expression from Step 2 to write *F* in terms of *m*, *R* and *T*.

Step 4: Use the formula from part 1 to express the force as

$$F_g = \frac{K}{r^2}$$

What is the value of K?

Solutions

Solutions

(1) let d_6 = distance from Earth to faintest object visible with Ground telescope.

B_6 = brightness of this faintest object

d = distance of an object of brightness $\frac{1}{50} B_6$.

Brightness varies inversely with the square of distance, so,

For some constant, K, $B_6 = \frac{K}{d_6^2}$ and $\frac{1}{50} B_6 = \frac{K}{d^2}$

$$\frac{1}{50}\left(\frac{K}{d_6^2}\right) = \frac{K}{d^2} \Rightarrow d^2 = 50 d_6^2 \Rightarrow d = 7.1 d_6$$

can see ~ 7 times farther

(2) $$(1 \text{ parsec})\left(\frac{3.09\times10^{13}\text{ km}}{1 \text{ parsec}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ sec}}{2.99\times10^{8}\text{ m}}\right)\left(\frac{1 \text{ hr}}{3600 \text{ sec}}\right)\left(\frac{1 \text{ day}}{24 \text{ hr}}\right)\left(\frac{1 \text{ year}}{365.25 \text{ days}}\right)$$

1 parsec = 3.27 light years.

(3) (a) $(10^9 \text{ parsec})\left(\frac{3.27 \text{ light years}}{1 \text{ parsec}}\right) = 3.27\times10^9$ years ago.

(b) It'll see 7 times farther which is, 2.3×10^{10} years ago.

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(1) $\frac{T^2}{t^2} = \frac{R^3}{r^3}$ so $T^2 = \left(\frac{t^2}{r^3}\right)R^3$ $\quad A = t^2/r^3$

$$v = \frac{d}{t} = \frac{2\pi r}{T}$$

$$F = \frac{mv^2}{R} = m\frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 mR}{T^2}$$

$$F = \frac{4\pi^2 mR}{T^2} = \frac{4\pi^2 mR}{(AR^3)} = \frac{4\pi^2 m}{AR^2} = \frac{K}{R^2}$$

where $K = \frac{4\pi^2 m}{A}$

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TAG NEEDS ADDRESSED

INTELLECTUALLY GIFTED

- ☒Advanced Critical Reasoning
- ❒Scholarly Interaction
- ☒Continuous Progress for Level and Rate*
- ❒Challenging Resources
- ❒Effecting Change
- ❒Decision Making; Ethical Use of Influence
- ❒Leadership Training/Career
- ❒Realistic Goal Setting
- ❒Regular Interaction with Intellectual Peers
- ❒Social-Emotional Issues; Support; Coping Strategies
- ❒Advanced Academic Planning
- ❒Opportunity for Competition/ Failures/Successes
- ❒Creative Problem Solving with Real Problems/Audience
- ❒Pursuit of Advanced Level Research
- ❒Advanced Vocabulary Development

ACADEMICALLY TALENTED MATH

- ☒Advanced Critical Thinking in Math
- ☒Continuous Progress/Level and Rate* in Math
- ❒Challenging Math Resources
- ❒ Creative Problem Solving Strategies in Math
- ❒Math Advanced Vocabulary Development
- ❒Leadership Training/Career
- ❒Decision Making; Ethical Use of Influence
- ❒Regular Interaction with Talented Math Peers
- ❒ Realistic Goal Setting
- ❒ Opportunity for Competition/Failures/ Successes
- ❒ Advanced Academic Planning in Math

* Rate requires monitoring to ensure that the student was allowed to move ahead upon acquiring concepts.

CAREER RELATED LEARNING STANDARDS FOR CAM - Certificate of Advanced Mastery

- ❒ Personal Management
- ☒ Problem Solving
- ❒ Communication
- ❒ Teamwork
- ❒ Employment Foundations
- ❒ Career Development

TEACHER CHECKS THE BENCHMARK LEVEL STUDENT

Math:

- ❒1
- ❒2
- ❒3
- ❒CIM
- ❒CAM

Student _____ **Grade** _____

Teacher _____ **School** _____

Date Initiated _____ **Date Completed** _____

Check TAG Identification category: ❒ **Intellectual** ❒ **Academic Math** ❒ **Academic LA**